

## ECONOMIC PHENOMENA VIA MATHEMATICAL MODELLING IN MAPLE SYSTEM

Zuzana Chvatalova<sup>1</sup>, Iveta Simberova<sup>2</sup>

Brno University of Technology, Kolejní 2906/4, CZE-612 00 Brno, Czech Republic  
E-mails: <sup>1</sup>[chvatalova@fbm.vutbr.cz](mailto:chvatalova@fbm.vutbr.cz); <sup>2</sup>[simberova@fbm.vutbr.cz](mailto:simberova@fbm.vutbr.cz) (corresponding author)

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**Abstract.** Nowadays, economics utilizes more and more frequently quantitative methods. This is an important phenomenon in the process of education. The paper deals with application of the Maple system for on-line creation of mathematical models in economic sectors. Means of Maple represent an important challenge for application in practice or research – resolve specific problems, apply mathematical models in many science fields, create intelligent documents for the presentation of modelling and analyses with computations, visualizations, animations, simulations. The paper incorporates selected samples of models in Maple and indicates further fields of its practical using, e.g. for measurement of marketing activity effectiveness.

**Keywords:** economics, Maple system, mathematical model, teaching, marketing research.

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**JEL classification:** A23, C88, M39.

### 1. Introduction

Economics is a social science. Its subject matter is society and behaviour of institutions and individuals of which it is composed. It is called a science to reflect the fact that, as a discipline, knowledge is built via the scientific method. This simply means that theories about phenomena in the economic domain are subjected to scrutiny that is both logical and empirical. Thus, economists are often called on by those in decision making positions, in government and the private sector, to provide economic input into the decision-making process (Griffiths 1993).

Mathematical simulation and modelling are unparalleled phenomena reflecting contemporary science disciplines. Mathematical modelling represents an integral part of the contemporary science. Simon and Blume (1994) wrote: “*Within the last thirty years, mathematics has emerged as the language of economics. Today economists view mathematics as an invaluable tool at all levels of study, ranging from the statistical expression of real-world trends to the development of fully abstract economic system*”.

This model can be understood as simplification of a real phenomenon. Modelling using the mathematical tool is named *mathematical modelling*. According to different aspects we can distinguish various types of models, such as verbal, visual, qualitative, analytical, physical, technical and intellectual, deterministic, stochastic, static, dynamic, etc.

The basis for economical analyses at the decision-making level is focused on mathematical modelling of real economical phenomena. Modelling can have miscellaneous features following from differing approach, application of numerous methods or availability of means.

Use of mathematical, numerical, statistical and econometric methods, optimization, operational analysis, fuzzy logic, neuron networks, genetic algorithms, catastrophe theory, etc., in mathematical modelling lead to *quantitative modelling* based on the so-called deep or shallow way. When formulating a real problem at the production stage, we have however primarily to provide *qualitative analysis* based also on intuition, reasonable estimation, own experiences and common-sense reasoning with the support of efficient interactive *information and communication technologies* (ICT).

Mathematical models used in scientific computing (Gander, Hřebíček 2004; Barnes, Fulford 2008) and economic computing (Simon, Blume 1994) are becoming large. Mathematical software for the *Maple system*, product of the Canadian company Maplesoft (<http://www.maplesoft.com>) is an appropriate ICT tool and computer environment enabling to solve such tasks.

## 2. Trends in teaching mathematical modelling

The requirements for teaching are closely linked with the needs of the current student generation, the communication and technical achievements and the various forms of studies (present, lifelong, e-learning, etc.).

In the age of the so-called “Net Generation” and widespread use of *social networking tools*, representing a significant part of the modern web (Web 2.0), the *Person-Centred Technology-enhanced Learning* (PCTeL) based on strong pedagogical foundations arising from the Person-Centred Approach (PCA) in learning shows to be an appropriate methodology reflecting the needs and expectations of the coming generation of students. It corresponds to what Stephen Downes (2005) understands as “E-learning 2.0”. Mathematics including calculus, linear algebra or differential equations, together with its application in mathematical modelling and simulation, traditionally understood as a hard and tedious discipline, creates at the same time a great opportunity to show the real strength of PCTeL (Hřebíček *et al.* 2010).

Currently, the development trend of ICT leads to *virtualization*. The Maple system creates a virtual environment grids and clouds (Chvatalova 2009).

To accept these challenges, teachers and students should be ready to observe the following rules in teaching mathematical modelling:

- support for both teamwork and autonomy and independence, particularly time and space independence;
- on-line learning, increased interactivity;
- provide technology and software background, virtualization and communication with social networking;
- interdisciplinary character of resolving problems;
- active participation in projects, research and in resolving specific problems of practice;
- co-operation (national and international).

Therefore the phase of education at universities (especially math education and computer literacy) plays an important role also in the context of national and international cooperation and relations. At the Brno University of Technology (BUT) students' internships abroad is encouraged as well as the teachers' mobility (teaching abroad, study visits, international projects) supported by the University itself, the Ministry of Education, Youth and Sports and by international institutions and projects. BUT study abroad can for example be realized in the following ways: Program LLP / Erasmus, Free Mover Scholarship EEA Norway, International Students Club (ISC), BEST Brno, Specific grants – House of International Services (DZS), Ministry of Education (<http://www.dzs.cz/>), the DAAD – Grants for German-speaking countries – (<http://www.daad.cz/>)<sup>1</sup>.

### 3. Mathematical modelling in economics using the Maple system

Maple environment is user friendly. It offers complex solutions to problems through an interactive mathematical modelling step by step (construction, analysis and modification of simple or nontrivial models of real economic phenomena, or specific models which need high performance computing, downloading large database sets, etc.). Computer algebra system Maple provides symbolic and numeric computations, advanced graphical 2D or 3D visualization, simulation and animation.

#### 3.1. Shortly about Maple system

In competition of the computer systems, supporting mathematic disciplines and their application in technical, social as well scientific sectors, the Maple system takes an important place. The system (Fig. 1) has been developing for over thirty years, nowadays by the Canadian company Maplesoft Inc. (<http://www.maplesoft.com>). Its successful development profile can be seen in the user popularity globally. Frequent dynamic and continuous innovations of the versions, responding to the latest ideas of the users, fields, sectors and practice, support success of its application in academic as well as in engineering, research and commercial sphere. The latest Maple 14 version was launched in spring 2010. Nowadays another version, Maple 15, is under progress.

<sup>1</sup> Available from Internet: <http://www.fbm.vutbr.cz/>.

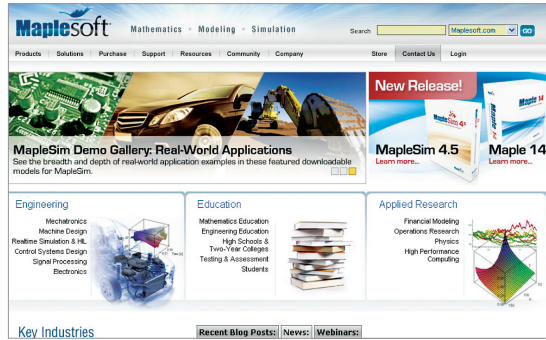


Fig. 1. Homepage of Maplesoft, Inc. – Maple 14 and MapleSim 4.5  
(Source: available from Internet: <<http://www.maplesoft.com/>>)

The *Czech Maple User Group* (<http://www.maplesoft.cz>), supporting cooperation of the Czech users by different activities and by organization of scientific forums, was established in the Czech Republic in 1993.

Significant Maple attributes are as follows:

- *complexity* of the system, based on support of application, documentation, communication and presentation activities of the users (own presentation code, structured Maple-worksheet and Maple-document with the possibility of commentaries, integrated references, space for personal share in guiding the issue to be resolved, support to feelings and intuition) – the so called *Maple Smart document* (Fig. 2);

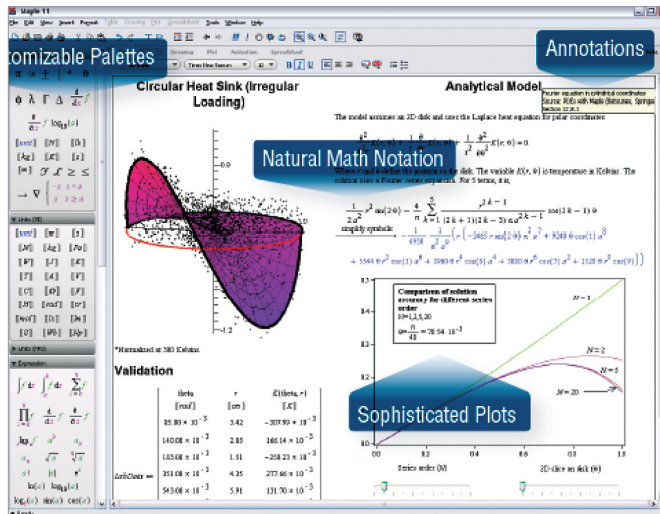


Fig. 2. Maple Smart document  
(Source: available from Internet: <<http://www.maplesoft.com/>>)

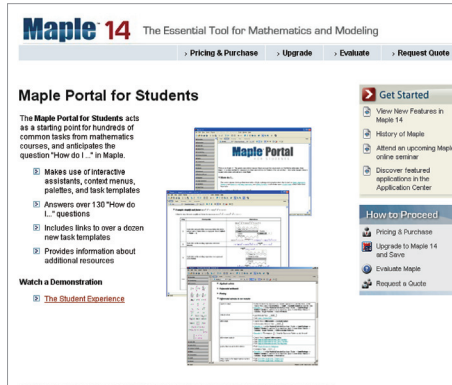
- support for *interactivity, exportability* (LaTeX, HTML, RTF, MathML), *compatibility* with other programming languages (Fortran, C, Java, Matlab, Vizual Basic, etc.), *possibility of presentation* of the “live” documents, regular on-line webinars (<http://www.maplesoft.webex.com>), open forums for the users (Maple Primes, Maple Connect Premier, Maple Connect, Maple conference), special programme for academic purposes – The Maple Adoption Program;
- comfort of the *working environment*, incorporation of interactive working elements and predefined working components, expendable tool ranges, clickable calculus, possibility of direct presentation (computations can be modified, interlaid by a stylized text, completed by references, numerical outputs can be formatted or converted automatically into the requested units, logic units and action sequences, analyses, computations and visualisations can be multi-layer structured);
- easy *system operation* by new users (Maple Help System – Ten Minute Tour, Maple Help tip, Quick Help) (Chvatalova 2008).

### 3.2. For students and teachers

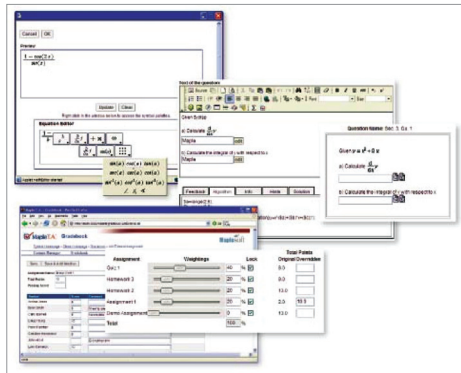
Mighty support for *universal mathematic education and application of mathematics in practice and research* is the guide line of the Maple system development. Besides the working environment for computations with a number of predefined functions, procedures and possibilities to program algorithmized steps (the programming language in Maple is similar to the Pascal language), the system also grants to the students the built-in Books and Study Guides, Dictionaries, Assistants, Tutors, Tasks, Demonstrations for remembering terms, definitions, claims and for training, for deeper and more comprehensible understanding of the analyzed issue in wider links, with the possibility of step-by-step explanation. *The Maple Media Center* has been established (<http://www.maplesoft.com/products/mapleta/features.aspx>). It contains videos – interactive demo-versions of procedures, live interviews and broadcasting with guests (Chvatalova 2008; Hrebicek, Chvatalova 2010a).

The *Student Help Center* (<http://www.maplesoft.com/studentcenter>) offers a Maple student forum, on-line mathematic Oracles, training videos and a mathematics homework resource guide. In the *Teacher Resource Center* (<http://www.maplesoft.com/teachercenter>) teachers can get the most out of your Maple teaching experience (Fig. 3). It provides sample applications, course material, training videos, white papers, e-books, podcasts, and tips (Hrebicek *et al.* 2010).

*Maple T.A<sup>TM</sup> Technology* is the potential for training content of the curriculum and for determination of clear rules for testing students' knowledge (Fig. 4). It offers high-quality means for classical teaching as well as for e-learning, training of knowledge and creation of tests. It is a real contribution for the students, instructors, tutors, guarantors and administrators (on-line training, testing, evaluation incl. creation of statistics of drawbacks or success, etc.) (Chvatalova 2008; Hrebicek, Chvatalova 2010a).



**Fig. 3.** Maple Portal for Students  
(Source: available from Internet: <<http://www.maplesoft.com/>>)



**Fig. 4.** Extract of documents for on-line testing and evaluation of the students by the Maple means  
(Source: available from Internet: <<http://www.maplesoft.com/>>)

The Maplesoft blog contains posts coming from the heart of Maplesoft. Find out what is coming next in the world of Maple, and get the best tips and tricks from the Maple experts. The Maple Portal is designed as a starting place for any Maple user. Maple's Tutorials will help you get started with Maple, learn about the key tools available in Maple, and lead you through a series of problems. From here, investigate more detailed topics in the Portals for Engineers, Students, and Math Educators. The *Clickable Mathematics* uses the *Maplet User Interface Customization* system that allows Maple users to create windows, dialogs, and other visual interfaces that interact with a user to provide the power of (Maple 2010).

### 3.3. Maple for practice and research

(A) Maple has also reached its user popularity in the practice. By its attributes Maple strengthens coexistence with the theory. The goal-directed structuring of the system,

its products and activities for characteristics groups of users, disciplines or spheres of issues belong to the important stimuli for solution and real tasks and problems of the practice.

Nowadays the following sectors can be highlighted: mechatronics, machine design, electronics, signal processing, control system design, real-time simulation & HIL (Maple 2010).

In this respect it has become an important priority for users of the *Application Center* (Fig. 5) system (<http://www.maplesoft.com/applications/>) where complete documents are concentrated and archived in Maple by specialists from all parts of the world, namely as approached by their authors – individual system users – both the users from practice and the users cooperating with academicians. The documents are author specified submitted in different formats. They can be assessed and in particular discussed – through web – both with the author himself and with all other users and specialists of the issue in question. This activity is important mainly for sharing mutual professional experience and views, free from any time or space barrier.

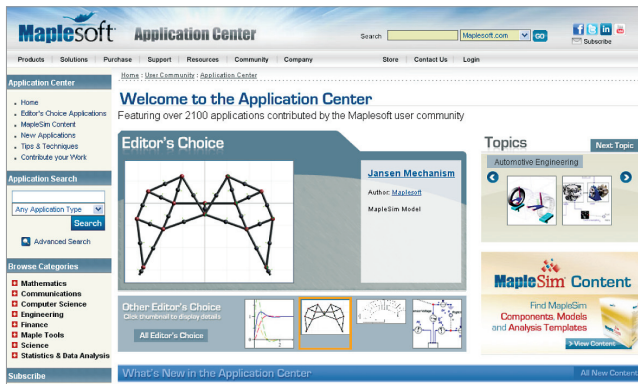


Fig. 5. Maple Application Center

(Source: available from Internet: <<http://www.maplesoft.com/applications/>>)

Let us note that the recently produced *MapleSim* is another important product, in particular for effective modelling, simulations and complex designing in the engineering practice. It is utilized both by individuals and by whole industrial branches. Its latest version MapleSim 4.5 becomes popular especially in the fields of technical practice. Its structural elements can also be applied in the sector of economic phenomenon simulation (Fig. 6).

With its core technologies of the world's most advanced symbolic computation engine and revolutionary physical modelling techniques, Maplesoft is uniquely positioned to offer cutting-edge software tools for design, modelling, and high-performance. It is an open secret that for instance Toyota, Bosch, Ford, Boeing, NASA, BMW, Canadian Space Agency, Canon, Motorola belong among customers of the company Maplesoft Inc. (Maple 2010).



Fig. 6. Maple Applied Research: Financial (Source: available from Internet: [http://www.maplesoft.com/applied\\_research/Financial/index.aspx](http://www.maplesoft.com/applied_research/Financial/index.aspx))

(B) Mathematics plays a vital role in most *scientific research and its applications*. We are in particular speaking about the fields of high performance financial modelling, operations research, computing, and physics. As the provider of the world’s most powerful mathematical computation engine, Maplesoft provides a strong foundation on which this work is based. The most natural and effective problem-solving approach is applied for your particular problem or specialty. In this respect we can name certain customers of the company Maplesoft Inc.: Bloomberg, Dreamworks, Bosch, NASAUS, Department of Energy National Security, Agency Jet Propulsion, Laboratory National, Institute of Standards and Technology, Bank of Canada, SLAC National Accelerator Laboratory (Maple 2010).

(C) Solution of practical and research issues is supported by separate *toolboxes* oriented on the issue in question, which are profiled step by step directly in Maple, e.g. Maple Financial Modelling, Toolbox Maple, Toolbox for Matlab, Global Optimization for Maple, Database Integration Toolbox, Maple-NAG® Connector, Professional Math Toolbox for LabVIEW, HPC-Grid Toolbox for Maple and others.

#### 4. Statistic means in Maple as support to marketing management

*Statistic means* in Maple, i.e. the built-in functions, procedures, 2D ad 3D visualization outfit, possibility of statistic diagnostics, means of regression analysis, work with data (even with collections having a high number of data), system linkability with other software platforms, possibility to use simple form elements, easy data exportability and importability from available office applications, etc., are understood the important basis



for complex assessment of *marketing research* in miscellaneous social spheres. Their effectiveness and quick application enables to compare different marketing methods, mechanisms, outputs of examination, communication of interested parties, direct result / outcome presentation, data archiving, and immediate modification of procedures, etc. Certain of these means have been applied for elaboration of the specific research which was realized on FBM BUT in the project “Measurement methods of marketing activities effectiveness and their application”<sup>2</sup>.

The *Statistics package* in Maple is a collection of tools for mathematical statistics and data analysis. The package supports a wide range of common statistical tasks such as quantitative and graphical data analysis, simulation, and curve fitting. Also included is the *Data Analysis Assistant*, a graphical interface to the data analysis tools in the *Statistics package*. It can be accessed from the *Tools > Assistants* menu or using the *InteractiveDataAnalysis* command (Maple 2010):

- In addition to standard data analysis tools the *Statistics package* provides a wide range of symbolic and numeric tools for computing with random variables. The package supports over forty major probability distributions and provides facilities for defining new distributions.
- Much of the functionality in the *Statistics package* is accessible through context menus. Context-sensitive functionality is available when selecting any data container (i.e. Vector, list, Array, etc.), known probability distributions (i.e. Normal (1, 2)), or random variables (Maple 2010).

The *Statistics package* is divided into the following parts (Maple 2010):

- *Inventory of Probability Distributions* (continuous and discrete distributions).

*For example:* Research activities very often apply the random variate governed by the normal (Gaussian) distribution (Cipra 2008). It is modelled by the Gauss curve (parameters  $\mu$  a  $\sigma$ ). The sequence of Maple commands:

> with (Statistics): X:=RandomVariable(Normal ( $\mu$ ,  $\sigma$ ));

$$> PDX(X, u) \quad \frac{1}{\sigma} \frac{e^{-\frac{1}{2} \frac{(u-\mu)^2}{\sigma^2}}}{\sqrt{\pi}}$$

$$> PDX(X, 0.5) \quad \frac{0.3989422802e^{-\frac{0.5000000000(0.5-1,\mu)^2}{\sigma^2}}}{\sigma}$$

$$> Mean(X) \quad \mu$$

$$> Variance(X) \quad \sigma^2$$

<sup>2</sup> Specific research of Faculty of Business and Management Brno University of Technology and was created as an one of theoretical results of the project “Measurement methods of marketing activities effectiveness and their application” reg. number FP-S-10-21 (2010). Supervisor is Dr Iveta Simberova, Ph.D.

- Descriptive Statistics, Data Summary and Tabulation.

*For example:* The descriptive statistics belongs to the basic methods of a number of researches. It often becomes an important instrument for decision-taking in economic disciplines (Rafael, Tvaronavičienė 2005; Oke, Oyedokun 2007; Ginevičius, Podvezko 2008; Ginevičius 2009). It is necessary to interpret correctly its outputs (statistic indicators) in conformity with the economic nature. The following sequence of commands in Maple highlights summary of statistics of the random variate governed by normal distribution with the parameters 10 and 3:

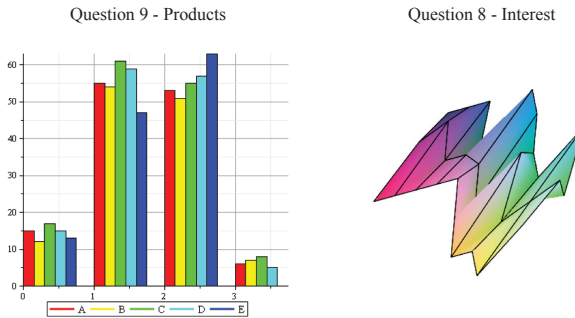
```
> with(Statistics): X := RandomVariable(Normal(10,3)): A := Sample(X, 10^4)
> DataSummary(A)
```

```
[mean = 9.969882096, standarddeviation = 3.000368661, skewness
= 0.01144367117, kurtosis = 2.978662089, minimum =
-1.468764996, maximum = 19.57590633, cumulativeweight
= 10000.]
```

- *Probability Calculations, Random Variables* (tools for creating and manipulating random variables as well as functions for computing their densities, moments, generating functions and other quantities (Giordano 2009). Here is the list of available commands).
- *Visualization* (various statistical plots).

*For example:* There are a number of 2D and 3D visualizations in Maple (Fig. 7). Let us show an example of the column graph for assessment of the closed control question that concerned consumer purchase of one of five products in four seasons. This question was the integral part of the marketing research questionnaire of a certain chain store. We can see that, using the bar graphs, it is possible to model the real answers is a well arranged form in Maple. Moreover, by applying the click calculus and the right mouse button, it is possible to modify the resultant visualization immediately by the needs which the graph shall serve for. The figure on the right side (surface plot) shows the 3D visualization of the stock in the store linked with material deliveries and sale.

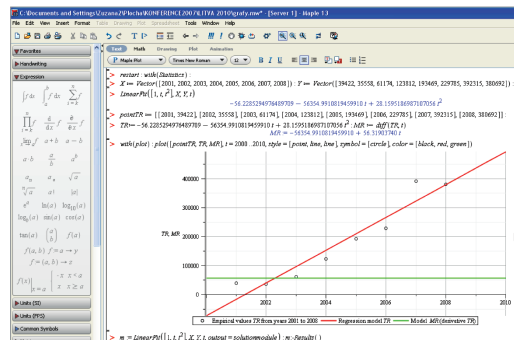
```
> restart : with(Statistics):
> ColumnGraph([[15,55,53,6], [12,54,51,7], [17,61,55,8], [15,59,57,5],[13,47,63,0]], color=red..blue,
gridlines, legend=["A", "B", "C", "D", "E"], title = "Question 9 - Products");
> restart : with(Statistics):
> X=(1, 2, 3, 0, 5, 4, 2, 1, 5, 1):Y=(2, 4, 2, 3, 3, 1, 5, 1, 3, 5):Z=(1, 2, 3, 4, 5, 3, 2, 1, 5):
> SurfacePlot(X, Y, Z, title="Question 8 - Intrest")
```



**Fig. 7.** ColumnGraph and SurfacePlot as visualization of marketing research outputs (Source: own work in Maple)

- *Simulation* (Maple supports optimized algorithms for simulating from all supported distributions as well as tools for creating custom random number generators, parametric and non-parametric bootstrap by the Statistics package).
- *Regression* (tools for fitting linear and nonlinear models to data points and performing regression analysis).

*For example:* When determining performance of the company, it is necessary to determine individual indications and ratios taken from financial statements of the company. By applying the regression analysis, it was possible to identify immediately development of the overall revenue (as the functional dependence) from the empirically established values of the period 2001 to 2008. It is the basic step for further analyses. It is for instance possible to analyze the revenue change rate by applying derivation of the established revenue function. Identification of other facts is offered as well. The revenue function maximum can be determined by the optimization means built in Maple (Fig. 8). Link to the demand enables to analyze demand flexibility. The regression model (revenue function) can also be subject to the statistic diagnostics which is predefined in Maple (Note: these facts are not shown here with respect to the scope of the paper).



**Fig. 8.** Maple document – Scatter plot of empirically acquired values and regression function – model of the overall revenue within the scope of analysis of company performance (Source: own work in Maple)

- *Estimation* (statistics package supports tools for manipulating likelihood functions, maximum likelihood estimation, kernel density estimation, bootstrap).
- *Data Manipulation* (tools for manipulating statistical data are also included in the Statistics package, e.g. Join, Sort, Rank, etc.).
- *Data Smoothing* (the Statistics supports many data smoothing functions including moving averages, exponential smoothing, linear filters, etc.).
- *Hypothesis Testing and Inference* (tools for performing hypothesis testing and inference, including several parametric and non-parametric tests are also in the Statistics package) (Maple 2010).

## 5. Chosen examples of economic phenomenon modelling in Maple

Students of the FBM BUT are led to study understanding of the economic theory as well as its applications in real practice (Hejc 2008; Hrebicek, Chvatalova 2009, 2010a, 2010b). They need to use ICT for the qualitative analysis of the real phenomena. Therefore, the form and the content of teaching mathematics and mathematical modelling are continuously innovated. In the year 2010, the FBM BUT with the financial support of the Ministry of Education realized the project<sup>3</sup> of the multimedia manual for teaching mathematics following *Clickable Mathematics* of Maplesoft. The multimedia manual is based on interactivity and capabilities of Maple in the modelling of economic phenomena to support decision making of company managers, outputs from research, etc. It is underpinned by Maple which provides a broad base of individual and teamwork for the future use in practice.

Solution of specific issues of specific companies, e.g. within the scope of elaboration of the final thesis or as direct participation in researches or projects, in their realization and assessment, is a great opportunity for the students how to apply the theoretical knowledge in practice. In these cases application of suitable computer software is an explicit guiding principle for success which is nowadays understood an inherent and inseparable feature which goes without saying.

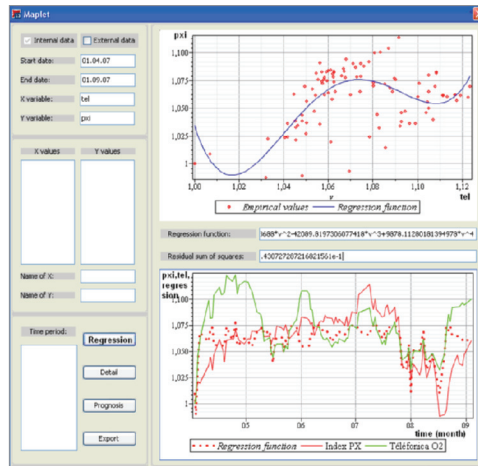
The following paragraphs will present chosen models of economic dependences supported and aided by the Maple system; in the major part we will present abstracts of outputs of the final theses of FBM BUT students and one abstract of the project<sup>3</sup>.

### 5.1. Mathematical modelling financial variables

Let us find the interdependence of the two time series of economic variables  $x_i, y_i$  ( $i = 1 \dots n$ ) that are empirically measured, e.g. at Stock Exchange. This is one of the basic tasks which perform in economics, finance and business.

<sup>3</sup> Project No. 3186/2010 “Videomanual as a multimedia support innovation in the subjects Mathematics and Mathematical seminar” of the Ministry of Education and Youth and it was also partly supported by FBM BUT. Supervisor Assistant Professor Dr Zuzana Chvatalova, Ph.D.

In the following maplet (Fig. 9) developed by (Zigardy 2010), the polynomial regression analysis is used for data downloading from the Stock Exchange. It also contains the visualization of fitting data by models. Maplet construction required the automatic choice of the degree of regression polynomial with the use of statistical methods, which must reflect numerous conditions based on the experience of economics, finance or business.



**Fig. 9.** Interactive Maplet – Scatter plot of empirically acquired values and regression function – model of the overall revenue within the scope of analysis of company performance (Source: own work in Maple)

Company managers can easily use this maplet for behaviour of the Stock Exchange. Its advantage is that it can easily upload time series in two ways: as the already downloaded data and stored data in maplet (Internal Data) or as the external sources (External data) by parsing data from the Stock Exchange. *Note:* The Maplet is designed to process the large collection of data, too. That is typical for example in a financial environment (such as Stock Exchange, etc.). We set a time period (in prescribed format xx.yy.zz) in maplet for the internal data (Start Date and End Date). We can set for the external data (Time period) their names of dependent and independent variables, which are used for labelling axes. The last toolbar on the left pane of maplet chooses output which we are interested in.

## 5.2. Mathematical modelling utility function

*Utility* belongs in economy among important topics. Utility is assessed as the method of description of the preferences considering the facts leading the consumers to certain decisions when purchasing a certain product. The utility function quantifies such situation (Varian 1995; Husek 2007; Konecny, Pitner 2010). Consumer preferences and their quantification by introduction of the utility function represent an important economic advantage connected with market description. Function of two variables (most frequently the so called Cobb-Douglas function) is considered a suitable utility model.

It can be visualized by 3D and 2D outputs by applying the indifferent curves and / or by determining other limits values, thus monitoring the dynamics of development. Fig. 10 presents the visualizations of the utility function  $U = U(x, y) = x^{\frac{1}{3}} \cdot y^{\frac{2}{3}}$  and the map of selected utility curves for levels  $U(x, y) = 2$ , resp. 4, resp. 6, resp. 8, resp. 10:

```

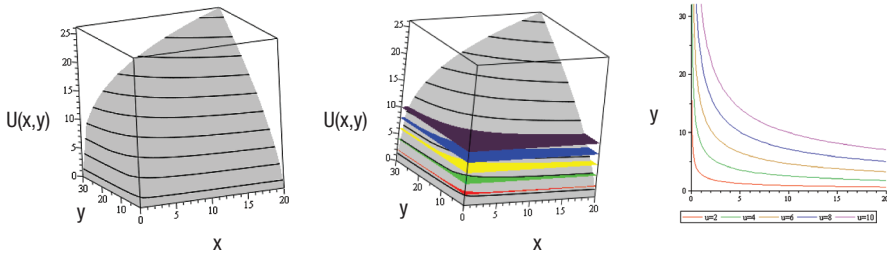
> U := (x,y) -> x^1/3 * y^2/3;    U := (x,y) -> x^1/3 * y^2/3

with(plots) : plot3d(U(x,y), x = 0 ..20, y = 0 ..30, color = gray, style
> = patch, axes = boxed, style = surfacecontour );

with(plots) : plot3d([U(x,y), 2, 4, 6, 8, 10], x = 0 ..20, y = 0 ..30, color
> = [gray, red, green, yellow, blue, violet], style = patch, axes
= boxed, style = surfacecontour );

> with(plots), U := (x,y) -> x^1/3 * y^2/3 :
> plot3d(U(x,y), x = 0 ..20, y = 0 ..30, color = gray, style = patch, axes = boxed, style = surfacecontour);
> plot3d([U(x,y), 2, 4, 6, 8, 10], x = 0 ..20, y = 0 ..30, color = [gray, red, green, yellow, blue, violet], style = patch, axes = boxed, style = surfacecontour);
> plot([ [ 2*sqrt(2)/sqrt(x), 8/sqrt(x), 6*sqrt(6)/sqrt(x), 8*sqrt(8)/sqrt(x), 10*sqrt(10)/sqrt(x) ], x = 0 ..20, y = 0 ..32, legend = ["u=2", "u=4", "u=6", "u=8", "u=10"];

```



**Fig. 10.** The exported visualizations of the utility function  $U(x,y)$ ; the cutting of selected planes, the map of selected utility curves from Maple document to MS Word document (Source: own work in Maple)

## 6. Conclusions

The latest research and teaching experience in the field of mathematic modelling and simulation of economic phenomena shows the rising complexity of issues and the process of education must respect new possibilities and needs arisen in research. Quick development of ICT means plays an important role for solution of problems, for analyses practical issues and interpretation of research results.

Teaching must be supported interactively by progressive modern tools. Number of useful features of the Maple system supporting mathematic modelling of economic phenomena has high-lighted in the paper. The incorporated means from the sector of statistics become the primary tools for application of researches for marketing and management of the companies.

For continuous development of the stock index, 3D model of the utility function and certain partial examples of economic variable dependence certain specific models constructed in Maple could be used (Maplet). To model and use more complex and less demanding tools for marketing efficiency measurements, namely both for the students and for marketing practitioners has enabled by the elaboration of certain metrics of measurement of marketing activity efficiency in the Maple system.

## Acknowledgement

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This work was also supported in part by the project No. 3186/2010 “Videomanual as a multimedia support innovation in the subjects Mathematics and Mathematical seminar” of the Ministry of Education and Youth and it was also partly supported by Faculty of Business and Management Brno University of Technology.

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## EKONOMINIŲ REIŠKINIŲ MATEMATINIS MODELIAVIMAS MAPLE SISTEMA

Z. Chvatalova, I. Simberova

Santrauka

Šiuolaikinė ekonomika vis dažniau ir plačiau taiko kiekybinius metodus. Tai yra svarbus veiksnys ugdymo procese. Straipsnyje nagrinėjamas *Maple* sistemos taikymas, matematinių modelių sukūrimas ir taikymas skirtinguose ekonominiuose sektoriuose. Jame apžvelgiamos plačios *Maple* sistemos taikymo praktikoje ir moksliniuose tyrimuose galimybės, sprendžiant konkrečias ekonomines problemas; matematinių modelių taikymo galimybės daugelyje mokslo sričių modeliuojant, analizuojant, kiekybiškai įvertinant ir vizualizuojant situacijas. Tokie modeliai turi platų taikomąjį pobūdį, pavyzdžiui, matuojant rinkodaros veiklos efektyvumą.

**Reikšminiai žodžiai:** ekonomika, *Maple* sistema, matematinis modelis, mokymas, rinkodaros tyrimai.

**Zuzana CHVATALOVA** is a scientist and university professor living in Brno. She graduated from the Masaryk University in Brno, Faculty of Science with the specialization in Mathematics. She received a Mgr. degree (Master of Science) in 1983 and RNDr. (Doctor of Science) in 1984. She received a Ph.D. in 2005 at the Brno University of Technology, Faculty of Business and Management. Since 1983 she has been working at the Brno University of Technology. She participates on many researches and regularly publishes in scientific journals.

**Iveta SIMBEROVA** is a PhD in Social Science and a Senior Lecturer in Marketing, and since 2004, she has combined this role with that of the Director of the Institute of Management, Brno University of Technology. Her research interests lie in modern marketing philosophy including total customer satisfaction, marketing culture and the internationalisation of small and medium enterprises. She worked as coordinator of several international projects and grants. In 2003, she defended her Dissertation Thesis with the title Use of Principles of Marketing Concept of Total Customer Satisfaction (TCS) for Company Development.

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